Hysteresis phenomena of the intelligent driver model for traffic flow

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We present hysteresis phenomena of the intelligent driver model for traffic flow in a circular one-lane roadway. We show that the microscopic structure of traffic flow is dependent on its initial state by plotting the fraction of congested vehicles over the density, which shows a typical hysteresis loop, and by investigating the trajectories of vehicles on the velocity-over-headway plane. We find that the trajectories of vehicles on the velocity-over-headway plane, which usually show a hysteresis loop, include multiple loops. We also point out the relations between these hysteresis loops and the congested jams or high-density clusters in traffic flow.

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Traffic jams are very annoying in daily life and have attracted considerable attention of the community of physicists for years [1]. Numerous studies of the dynamics of traffic jams have accumulated over the decades. Most of them study the transition from free flow to the congested state or the dynamics of relaxation of congested traffic flow [2,3]. The traffic flow is a complex system containing many particles with strong interactions, so it certainly shows some complexities such as self-organization and initial sensitivity. However, there is little research concerning the effects of the initial state on traffic flow. Therefore, we concentrate on this problem and explore the dynamics of traffic flow.

The dynamics of traffic flow have been discussed in a variety of traffic models, such as car-following models, cellular automaton models, and hydrodynamic models [4]. Recently, the most widely used car-following model is the optimal velocity model proposed by Bando *et al.* [5]. By this kind of optimal velocity model, the effects of fluctuations of the leading car and the explicit delay time in the formation of traffic jams are analyzed, and the hysteresis loop of a single vehicle is introduced to describe the motion of cars and the dynamics of fully developed jams [6–9]. The bifurcation phenomena of optimal velocity model are also explored [4,10]. However, the optimal velocity model does not contain the driver's response to the relative velocity with respect to the leading car, and collision accidents could not be avoided in this model [11]. So Treiber and co-workers proposed an intelligent driver model [12–15], which is still a type of car-following model including the response of the driver to the relative velocity with respect to the leading car. The intelligent driver model is more closely approximate to what drivers actually do and is easy to calibrate, accident free, and reproduces the empirically observed phenomena [1]. This model also has an equivalent macroscopic version called the nonlocal, gas-kinetic-based traffic model [1,13]. Here we adopt an intelligent driver model to explore the effects of the initial state on traffic flow and use the microscopic diagram of vehicles to study the structure of traffic flow.

As shown in Fig. 1, we consider N vehicles running on a circular one-lane roadway of length L. The driver of vehicle α continuously changes velocity according to its own velocity v_α, headway (netto distance) $s_\alpha = x_{\alpha-1} - x_\alpha - l_\alpha$, and relative velocity with respect to its leading car, $\Delta v_\alpha = v_\alpha - v_{\alpha-1}$, as follows:

$$\frac{dv_\alpha}{dt} = a_\alpha\left[1 - \left(\frac{v_\alpha}{v_{0,\alpha}}\right)^\delta - \left(\frac{\bar{s}(v_\alpha,\Delta v_\alpha)}{s_\alpha}\right)^2\right], \tag{1}$$

where a_α is the maximum acceleration, x_α is the coordinates of vehicle α, l_α is the length of vehicle α, $v_{0,\alpha}$ is the desired velocity of vehicle α, s_α is the actual netto distance, δ is the acceleration exponent, and $\bar{s}(v_\alpha,\Delta v_\alpha)$ is the minimum desired netto distance which varies dynamically with the velocity and relative velocity:

$$\bar{s}(v,\Delta v_\alpha) = s_{0,\alpha} + s_{1,\alpha}\sqrt{\frac{v}{v_{0,\alpha}}} + T_\alpha v + \frac{v\Delta v_\alpha}{2\sqrt{a_\alpha b_\alpha}}, \tag{2}$$

where s_0 and $s_{1,\alpha}$ are the jam distances describing the gap between cars in a traffic jam, T_α is the safe time headway, and b_α is the desired deceleration. In the rest, we explore the case of identical cars with the following parameters by numeric simulation: $v_{0,\alpha}=v_0=20$ m/s, $s_{0,\alpha}=s_0=1.5$ m, $s_{1,\alpha}=s_1=0$ m, $T_\alpha=T=1.2$ s, $a_\alpha=a=0.8$ m/s^2, $b_\alpha=b=1.8$ m/s^2, $l_\alpha=l=5$ m, and $\delta=4$. The simulation step is equal to 0.1 s.

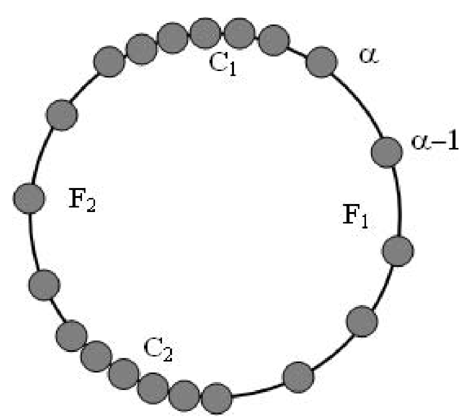

FIG. 1. Sketch map of traffic flow in a circular one-lane roadway. There are two congested jams C_1 and C_2 and two free flows F_1 and F_2. There are n_1 and n_2 vehicles in F_1 and F_2. The length of the free flow is s_1 and s_2, respectively. Vehicle α departs from C_1 and changes its speed according to its own velocity v_α, headway, and the relative velocity with respect to its leading car, $v_{\alpha-1}$.

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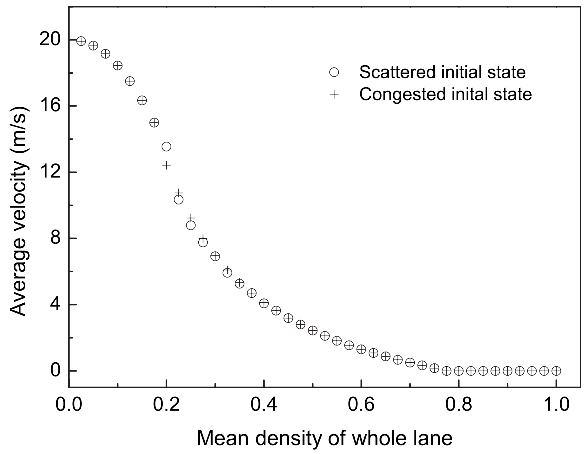

FIG. 2. The average velocity of final traffic flow over the mean density $\rho=\frac{Nl}{L}$, where $N=150$, and the length of lane L ranges from 750 to 30 000 m. The simulation results of 3×10^5 steps are identical to that of 3×10^6 steps. Although the system starts from totally different initial states, the average velocities at every density are approximately the same as each other, which are also approximately same as that of the nonlocal, gas-kinetic-based traffic model's result.

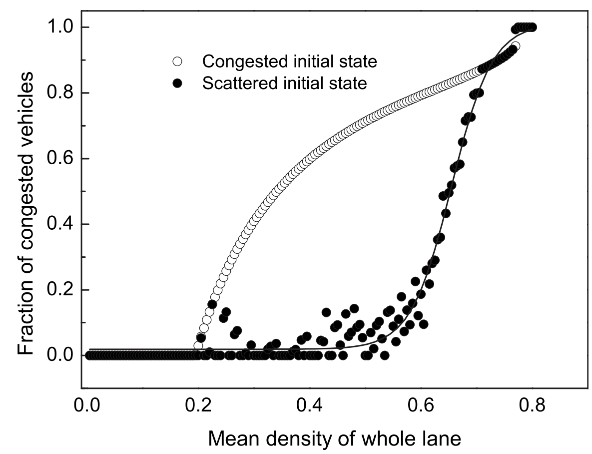

FIG. 3. The fraction of congested vehicles (η) over density, where $N=150$, and the length of lane range from 750 to 30 000 m. The simulation results of 3×10^5 steps are the same as that of 3×10^6 steps, where the solid line is the sigmoidal fit to the fraction of congested vehicles formed by the scattered initial state with $R^2=0.98719$. The results show a typical hysteresis loop and imply that the initial state has an important effect on the microscopic structure of traffic flow.

Without loss of generality, we consider the effects of two kinds of initial conditions, which are called the congested initial condition and the scattered initial condition, on the dynamics of traffic flow. The congested initial condition is that all vehicles are set in one large cluster with zero netto distance and zero initial velocity. On the contrary, the scattered initial condition is that all the vehicles are positioned in the lane with the same netto distance $s=\frac{L}{N}-l$ and their velocities range from 0 to 1 m/s randomly.

Figure 2 shows that the average velocities of traffic flows almost equal each other. Nevertheless, they evolved from a scattered or congested initial state with the same density. But the microscopic structures of the traffic flows are entirely different as shown in Fig. 3, whose vertical axis is the fraction of congested vehicles denoted by $\eta=\frac{n}{N}$, where n is the total number of congested vehicles in jams and the horizontal axis is the mean density of the whole lane. Figure 3 shows a typical hysteresis loop, which implies that the number of congested vehicles in the traffic flows is different if their initial state is different. Given $\rho<\rho_{c1}\sim0.2$, the initial congested vehicles completely dissolve and the free flow cannot form any traffic jams. When the density satisfies the condition $0.2\sim\rho_{c1}<\rho<\rho_{c2}\sim0.5$, the initial congested vehicles cannot completely dissolve and form one moving extended traffic jam with η_c, but the scattered initial state will not develop into traffic jams spontaneously. If the density is larger than ρ_{c2} and smaller than $\rho_{c3}\sim0.715$, the traffic jam spontaneously emerges from the scattered initial state which is described as a second-order phase transition in Refs. [1,12,13]. And the fraction of congested vehicles, η_s, from the scattered initial state is smaller than that of the congested initial state—i.e., $\eta_s<\eta_c$. When the density is $\rho_{c3}<\rho<\rho_{c4}\sim0.78$, the different initial states lead to the same fraction of congested vehicles, while, if the density is larger than ρ_{c4}, all the vehicles are in jams, for the effect of jam distance $s_{0,\alpha}$.

The results in Fig. 3 give us the main conclusion that the initial state has an important influence on the structure of the traffic flow produced by the intelligent driver model. However, η in Fig. 3 is an average quantity and cannot reveal the elaborate structure of traffic flow. So we draw the trajectories of each vehicle on the time-over-location plane as shown in Fig. 4. Given two moderate densities $\rho=0.35$ and $\rho=0.65$, the diagrams indicate that congested initial state induces only one large backward-moving traffic jam, whereas the scattered initial state could lead to several small backward-moving traffic jams, which causes the difference between η_c and η_s.

In order to quantitatively lay out the difference of traffic flows developed from congested and scattered initial states, we draw the trajectories of vehicles on the velocity-over-headway plane, which exhibits the time course that vehicles leave and get into high-density clusters or congested jams clearly [9]. Due to the simple structure, we investigate the behavior of traffic flow from the congested initial state at first (see Fig. 5). Given that the density is larger than $\rho_{c4}\sim0.78$, the trajectory is one point with zero velocity, which means that no vehicle can move due to the jam distance $s_{0,\alpha}$. When the density ranges from $\rho_{c2}\sim0.2$ to $\rho_{c4}\sim0.78$, the trajectory forms one hysteresis loop with zero-velocity point, which implies that the congested jam has not dissolved completely. Combining the fact that the congested fraction is larger than zero in Fig. 3 with the fact that hysteresis loops do not change from 3×10^5 to 3×10^6 simulation steps, it can be drawn that the congested jam is stable. In this case, the vehicle departs from the congested jam at first, gets into the free-driving region, and then enters the same backward-moving jam, which is similar to the time course of fully developed jams in Ref. [9]. Additionally, the smaller the density is, the larger the loop is, which indicates that more congested vehicles dissolve and the maximum speed becomes

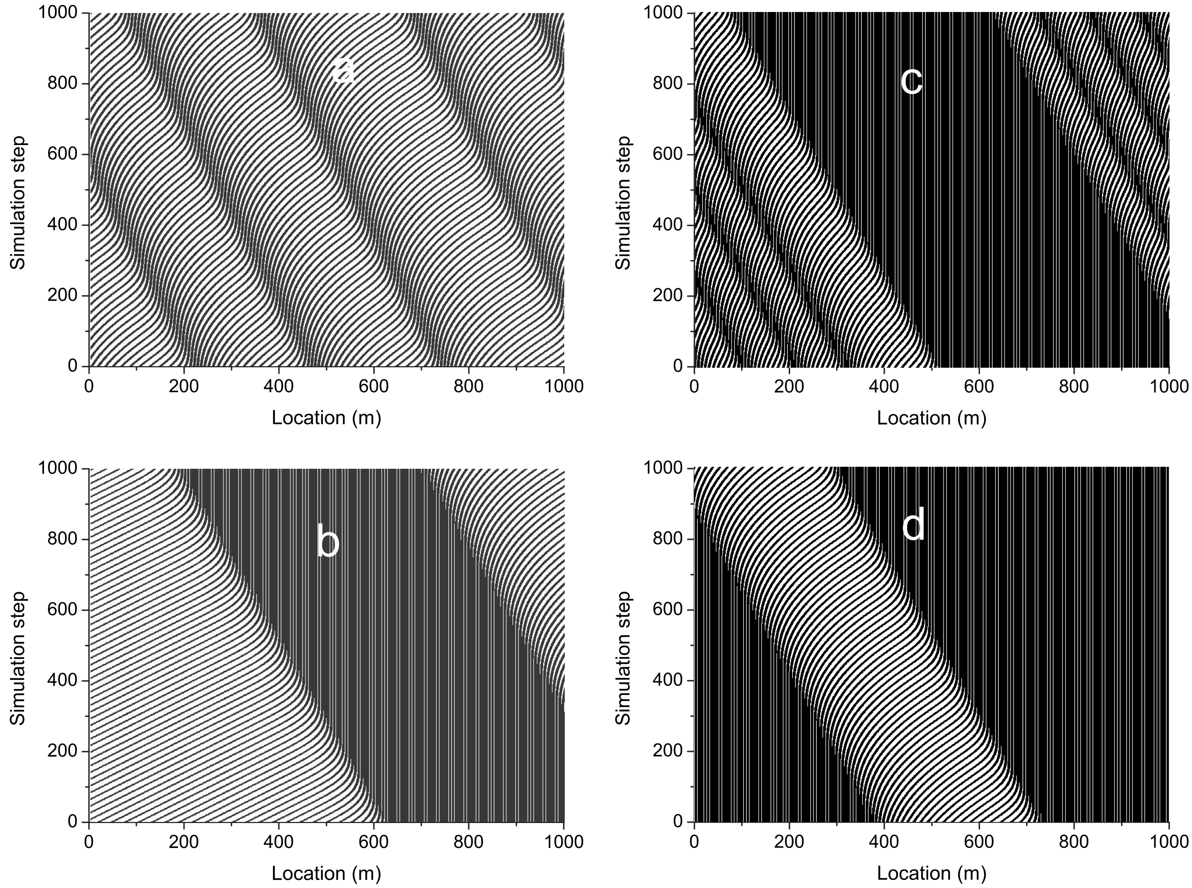

FIG. 4. Trajectories of each vehicle on the time-over-location plane, $N=150$, in the first 1000 m of the circular one-lane and 1000 steps after 3×10^5 simulation steps. The Density in (a) and (b) is 0.35 ($\rho_{c1}<\rho<\rho_{c2}$) and that in (c) and (d) is 0.65 ($\rho_{c2}<\rho<\rho_{c3}$). (a) and (c) describe the traffic flow from scattered initial state, which include several high-density clusters or jams. (b) and (d) describe the traffic flow from congested initial states, which include only one large congested jam.

larger while the density decreases. When the density is smaller than $\rho_{c1}\sim0.2$ and larger than $\rho_{c0}\sim0.17$, the trajectory is one hysteresis loop without a zero-velocity point, and the congested jam entirely dissolves with a high-density cluster region in the traffic flow. The trajectory shrinks to one

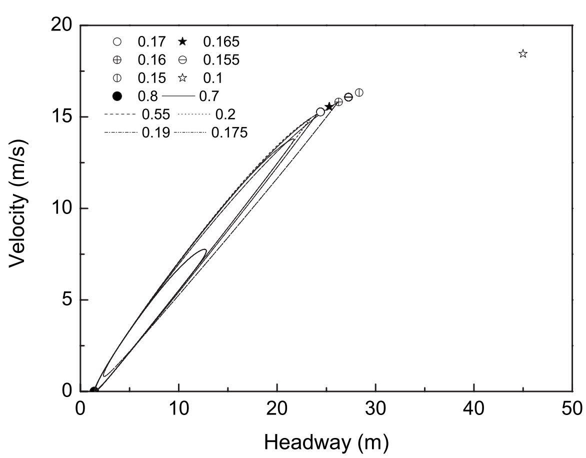

FIG. 5. Hysteresis loops on the velocity-over-headway plane induced by the congested initial state with different densities. Since the hysteresis loops do not change from 3×10^5 to 3×10^6 simulation steps, these loops are stable.

point again when the density becomes further smaller and all the vehicles are running with approximately equal netto distance at the same velocity. If the traffic flow becomes sparsely enough, the trajectory becomes into one line, which means that all vehicles are not constrained by the leading vehicle. The hysteresis loops of vehicles exhibit the relaxation process of a congested traffic jam: the jam partially dissolves at high density; the jam completely dissolves with high-density clusters at a lower density; the jam completely dissolves and the vehicles move constrained by the leading car; the congested jam utterly dissolves and vehicles run freely.

Now, we investigate the hysteresis loop of traffic flow from the scattered initial state. When the density is larger than ρ_{c3}, the hysteresis loop from the scattered initial state overlaps that from the congested initial state [see Fig. 6(a)], which is apparently consistent with the result of Fig. 3. If the density is smaller than ρ_{c1}, the hysteresis loop shrinks to one point, which means that all vehicles are running without high-density and sparse waves. When the density ranges from ρ_{c1} to ρ_{c3}, we find an interesting phenomenon, which has not been reported in the previous literature and is quite different from the results of the optimal velocity model, the trajectories of vehicle on the velocity-over-headway plane are multiple loops [see Figs. 6(b) and 6(c)]. These multiple

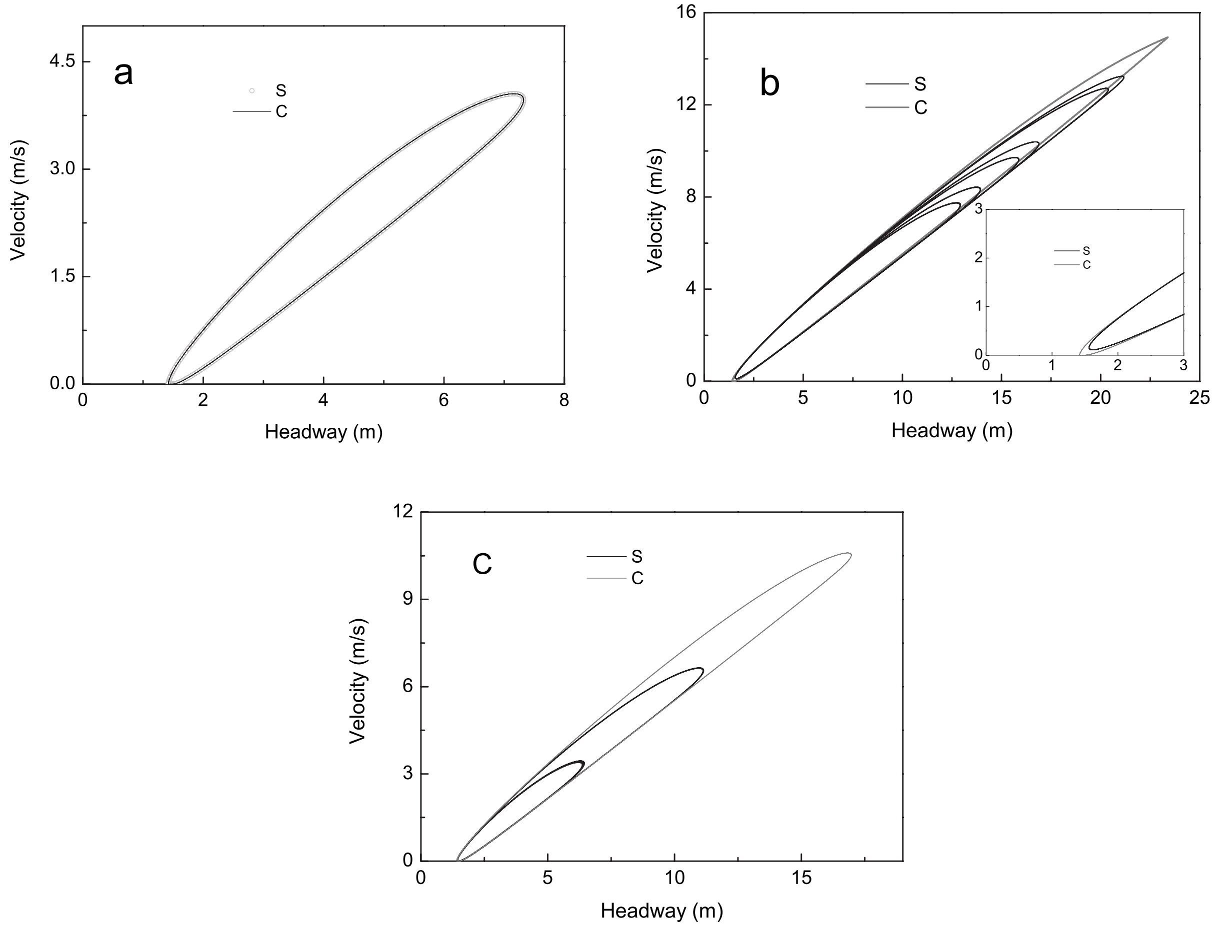

FIG. 6. Hysteresis loops at different densities on the velocity-over-headway plane. *S* corresponds to the scattered initial state and C to the congested initial state. (a) $\rho=0.75$. There is only one hysteresis loop, which is the overlapped hysteresis loop from different initial state. (b) $\rho=0.35$. The congested initial state leads to one loop with a zero-velocity point, whereas the scattered initial state induces multiple loops without a zero-velocity point (see the inner graph). (c) $\rho=0.65$. The congested initial state leads to one hysteresis loop, and the scattered initial state results in multiple loops with zero-velocity points. The multiple hysteresis loops in (b) and (c) are transient results of traffic flow induced by the scattered initial condition; the simulation step is 3×10^5.

loops mean that one can drive at different speeds and even the headway or density is same. This phenomenon can explain what happens in reality; for instance, the following car can move at a high speed with small headway if the velocity of the leading car is high.

Actually, the hysteresis loop reflects the microscopic structure of traffic flow. In order to make sure of this point, we explore the one loop first. In fact, each hysteresis loop exhibits four states of the vehicle's motion: staying on the jam with minimum speed v_{min}, getting away from the jam to free flow and acceleration, moving at maximum speed v_{max}, deceleration and getting into a jam. If the traffic flow has high-density clusters without a congested jam, we can get $v_{min}>0$; otherwise, $v_{min}=0$. Once the vehicle departs from the jam, it speeds up until it reaches the maximum velocity v_{max} which is determined by the actual headway and relative velocity with respect to its leading car. The vehicle cannot maintain the maximum speed for a period. To illustrate this point, we assume that some vehicles move at v_{max} in time interval $[t_0,t_1]$ and we can get $v_{\alpha-1}=v_{max}-(v_{max}-v_{\alpha-1,t_0})e^{\beta(t-t_0)}$ (β is constant) for the intelligent driver model and $x_{\alpha-1}=x_\alpha+d$ (d is constant related to v_{max}) for the optimal velocity model. However, these formulas cannot hold during whole interval $[t_0,t_1]$ [16]. Then, the vehicle slows down immediately until its speed decreases to v_{min}. As a consequence, the same moving process of vehicles in the same free flow corresponds to the same hysteresis loop, and different moving processes of vehicles in different free flow causes different hysteresis loops on the velocity-over-headway plane. At the same time, the same free flow leads to the same moving process of vehicles since all vehicles obey the same rule and start from v_{min} in the traffic flow. Therefore, the multiple hysteresis loops indicate that there are several different free flow in the traffic flow.

Before the traffic flow reaches its steady state, the congested jams or high-density clusters will vary along with the evolution of traffic flow, which results in a change of the free flow in the traffic flow and hysteresis loops on the velocity-over-headway plane. As far as the optimal velocity model is concerned, the hysteresis loops for transient traffic flow change drastically and look like multiple loops because of a short relax time, and there is only one hysteresis loop for the

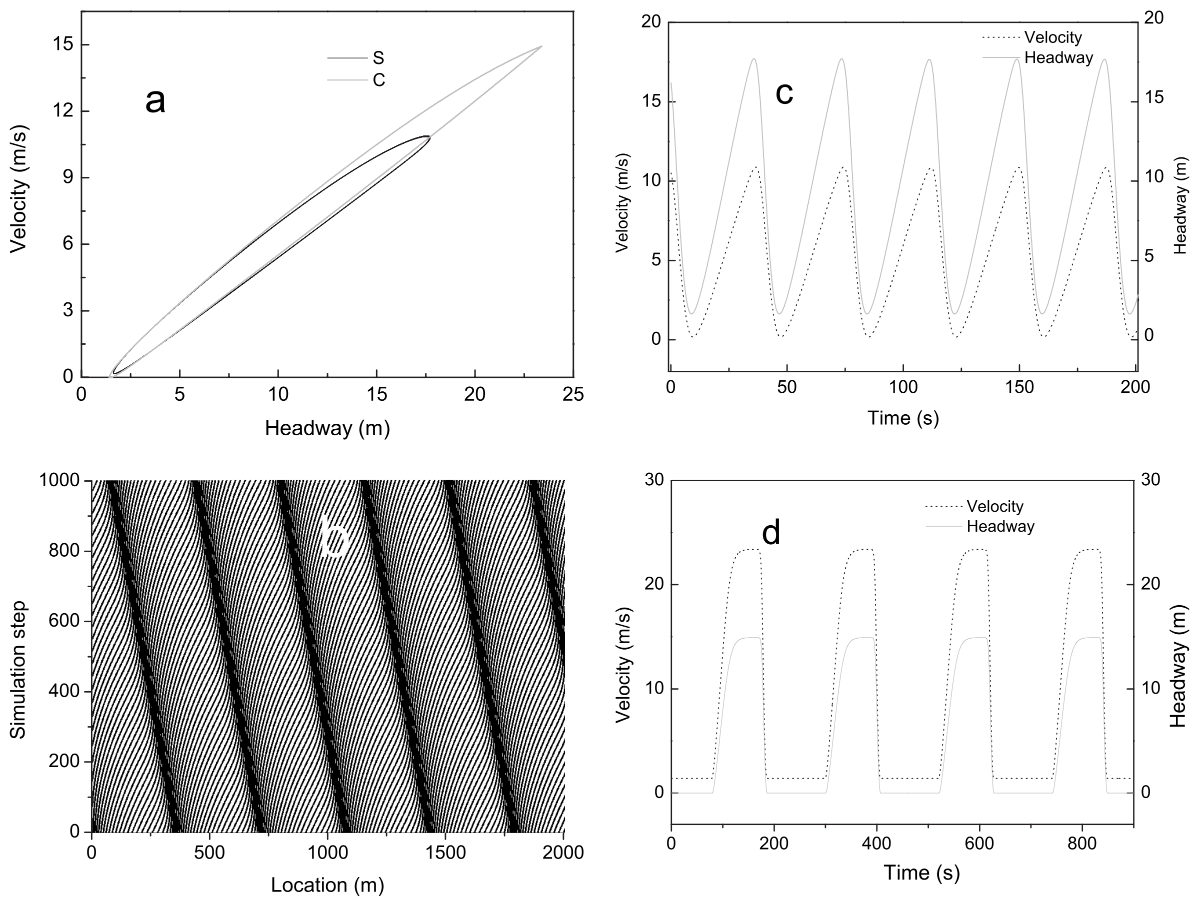

FIG. 7. Multiple hysteresis loops in Fig. 6(b) merge into one hysteresis loop after 3×10^7 simulation steps which is about 830 h or 30 days (upper graph). And the traffic flow includes several completely free-flow high-density clusters (b). Plots of velocity and headway against time are shown in (c) and (d), which are similar with that of optimal velocity model [17]. (c) is the results from the scattered initial state and (d) is from the congested initial state.

final steady state. Concerning the case of the intelligent driver model, there are observable multiple hysteresis loops in the transient traffic flow because of the long time of relaxation. These transient multiple hysteresis loops may merge into one stable hysteresis loop or several different stable hysteresis loops through a very long relaxation time. Actually, the multiple hysteresis loops in Fig. 6(b) merge into one hysteresis loop after about 3×10^7 simulation steps relaxation. The final traffic flow from the scattered initial state has several completely the same congested jams and free flow, whereas traffic flow from the congested initial state has only one larger congested jam and longer free flow; therefore, the hysteresis loop of traffic flow from the scattered initial state is smaller than that from the congested initial state as shown in Figs. 7(a) and 7(b). Figures 7(c) and 7(d) show the results of velocity and headway against time from the scattered initial state and congested initial state, which are similar with that of the optimal velocity model and there is a series completely the same pulses. The results mean that there is a series of completely the same jams or only one back-moving jam in the road. After about 10^9 steps simulation, the traffic flow from the scattered initial state with density 0.65 evolves into its steady state. The multiple hysteresis loops in Fig. 6(c) do not merge into one hysteresis loop but three different hysteresis loops in the steady state of traffic flow as shown in Fig. 8. These three different hysteresis loops correspond to three different congested jams. The plot of velocity and headway over time shows three different pulses in one period, which is quite different from that of the optimal velocity model. In this steady state of traffic flow, every different congested jam will move at the same velocity. Otherwise, the jams will merge into larger jams, and the number of vehicles in any congested jams is maintained as constant which means that the number of vehicles that depart from some jam is equal to that to get into the same jam in any time interval. For example, the average number of vehicles entering congested jam C_2 in Fig. 1 in the unit time interval equals $\rho_1\overline{v_1}=\frac{n_1}{s_1}\frac{1}{n_1}\Sigma_{j\in F_1}v_j=\frac{1}{s_1}\Sigma_{j\in F_1}v_j$ and the average number of vehicles leaving C_2 in the unit time interval equals $\rho_2\overline{v_2}=\frac{1}{s_2}\Sigma_{j\in F_2}v_j$. If Fig. 1 shows some steady state of traffic flow, the following formula holds:

$$\frac{1}{s_1}\sum_{\alpha\in F_1}v_\alpha=\frac{1}{s_2}\sum_{\alpha\in F_2}v_\alpha. \tag{3}$$

If the congested jams are more than those in Fig. 1, the above formula changes into the following:

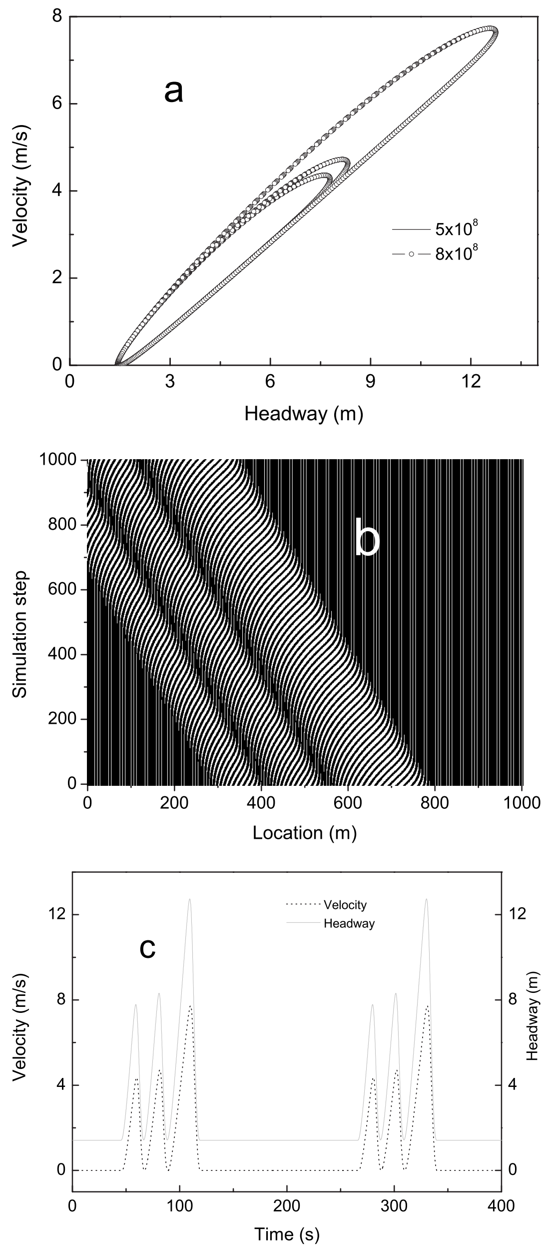

FIG. 8. Multiple hysteresis loops in Fig. 6(c) evolve into their steady state after about 10^9 simulation steps. (a) The results of 5×10^9 and 8×10^9 overlapped. (b) The traffic flow includes several different free flow and congested jams. (c) Plot of velocity and headway over time include three different pulses, which indicates that there are three different jams in the road.

$$\frac{1}{s_1}\sum_{\alpha\in F_1} v_\alpha = \frac{1}{s_2}\sum_{\alpha\in F_2} v_\alpha = \cdots = \frac{1}{s_j}\sum_{\alpha\in F_j} v_\alpha . \tag{4}$$

Actually, the final steady state of traffic flow in Fig. 8 abides the above formula. According to the simulation results, the quantities of different free flow in the steady state of traffic flow are 0.26 ± 0.03, 0.25 ± 0.02, and 0.26 ± 0.02, respectively. In a brief word, in the case of the scattered initial state, there are multiple hysteresis loops in the transient of traffic flow, and these multiple hysteresis loops may converge into only one hysteresis loop or several different hysteresis loops in the final state of traffic flow.

In this paper, we report the hysteresis phenomena of the intelligent driver model and find that the microscopic structure of traffic flow produced by the intelligent driver model is dependent on its initial state. It is observed that traffic flow from the scattered initial state could experience different scenarios as the density increases: free flow without density wave, traffic flow with several high-density clusters (multiple hysteresis loops without zero-velocity point), traffic flow with several different congested jams (multiple hysteresis loops with zero-velocity point), traffic flow with one or several completely same free flow (single loop with zero-velocity point), and all vehicles congested. On the contrary, to relax that one pinned localized cluster, which is also called the congested initial condition here, the traffic flow undergoes another route as the density decreases: all vehicles congested, traffic flow with a solo congested jam, traffic flow with a solo high-density cluster, proportionally distributed traffic flow, and free traffic flow.

Comparing with the results mentioned above, the fraction of congested vehicles of optimal velocity model is independent on the initial state of traffic flow, and the trajectories of vehicles have only one hysteresis loop on the velocity-over-headway plane [18]. It is obvious that the difference is caused by the hypothesis about the acceleration of the two models. From the simulation results in this paper, the hypothesis of the intelligent driver model is closer to the facts in the real world.

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[1] D. Helbing, Rev. Mod. Phys. **73**, 1067 (2001).
[2] M. Gerwinski and J. Krug, Phys. Rev. E **60**, 188 (1999).
[3] N. Moussa, Phys. Rev. E **71**, 026124 (2005).
[4] Y. Igarashi, K. Itoh, K. Nakanishi, K. Ogura, and K. Yokokawa, Phys. Rev. E **64**, 047102 (2001).
[5] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, Phys. Rev. E **51**, 1035 (1995).
[6] M. Bando, K. Hasebe, K. Nakanishi, and A. Nakayama, Phys. Rev. E **58**, 5429 (1998).
[7] H. Hayakawa and K. Nakanishi, Phys. Rev. E **57**, 3839 (1998).
[8] T. Nagatani, Phys. Rev. E **61**, 3534 (2000).
[9] Y. Sugiyama and H. Yamada, Phys. Rev. E **55**, 7749 (1997).
[10] G. Orosz, R. E. Wilson, and B. Krauskopf, Phys. Rev. E **70**,

026207 (2004).

[11] D. Helbing and B. Tilch, Phys. Rev. E **58**, 133 (1998).

[12] M. Treiber and D. Helbing, J. Phys. A **32**, L17 (1999).

[13] M. Treiber, A. Hennecke, and D. Helbing, Phys. Rev. E **62**, 1805 (2000).

[14] D. Helbing, A. Hennecke, and M. Treiber, Phys. Rev. Lett. **82**, 4360 (1999).

[15] A. Hennecke, M. Treiber, and D. Helbing, in *Traffic and Granular Flow '99*, edited by D. Helbing, H. J. Herrmann, M. Schrekenberg, and D. E. Wolf (Springer, Berlin, 1999).

[16] Here we describe the theroetical results; however, the simulation results indicate that a constant speed can be maintained for a period for optimal velocity model such as in [6], which is caused by the limitation of the simulation precision. Therefore, the different free flow could correspend to the same hysteresis loop if they have same acceleration and decceleration process.

[17] G. Orosz, B. Krauskopf, and R. E. Wilson, Physica D **211**, 277 (2006).

[18] Here we use the optimal velocity model of Ref. [3] to operate all homogeneous vehicles in a circular one-lane roadway. There are 100 cars in traffic flow. The length of roadway ranges from 600 to 3000 m. The length of each vehicle is $l=5$ m. And we adopt two different groups of parameters: $\alpha=2$, $\tau=0$ s and $\alpha=2.8$, $\tau=0$ s.